Radar backscattering properties of nonspherical ice crystals at 94 GHz

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The millimeter wavelength radar backscattering properties at 94 GHz for six nonspherical ice crystals, which include hexagonal column, hollow, plate, bullet rosette, aggregate, and droxtal with 46 maximum dimensions ranging from 2 to 10,500 μm, are investigated using the discrete dipole approximation (DDA) method and Lorenz-Mie theory. It is found that the radar backscattering cross sections are sensitive to ice crystal habits and their representations, which use spherical particles with equivalent maximum dimension, volume, projected area, or the ratio of volume to projected area to model nonspherical ice crystals for the Lorenz-Mie theory. The radar backscattering cross sections of the six nonspherical ice crystals from the DDA method are further parameterized as functions of maximum dimensions of ice crystals. The results from the parameterizations agree well with those computed from the DDA method. Moreover, the mean radar backscattering cross sections derived by averaging the results from the parameterizations over gamma distributions for ice clouds are consistent with those from the DDA method. The parameterizations are applied to derive the coefficients for the relationships between equivalent radar reflectivity factor and ice water content for ice clouds consisting of the six nonspherical ice crystal habits or a mixture of habits. The coefficients from the parameterizations are close to those from the DDA method. Both of them are sensitive to ice crystal habits. The pronounced differences among the relationship between equivalent radar reflectivity factor and ice water content for an ice cloud with a mixture of ice crystal habits, which has been extensively used for solar and infrared retrievals of ice cloud properties, and several previous relationships further confirm their sensitivity to microphysical properties of ice clouds.


1. Introduction

[2] Ice clouds, which are globally distributed, are still a source of major uncertainties in satellite-based retrievals and climate-modeling studies [e.g., Liou, 1986; Stephens et al., 1990; Lynch et al., 2002]. They play an important role in the climate system through their effects on the radiative balance of the Earth-atmosphere system [Lynch et al., 2002]. It has been found that the radiation forcing of ice clouds is sensitive to their optical and microphysical properties [Stephens et al., 1990; McFarquhar et al., 2002]. Passive infrared, visible, and microwave techniques have been used for remotely sensing optical and microphysical properties of ice clouds [Lynch et al., 2002]. Amongst them millimeter wavelength radars have an advantage of detecting the dynamical and structural properties of ice clouds [Aydin and Tang, 1997], so their backscattering properties have been used to estimate ice cloud properties [e.g., Sassen et al., 2002]. The knowledge of radar backscattering properties of ice clouds is the basis for developing the relationships between radar reflectivity factor and ice cloud properties [e.g., Sassen et al., 2002].

[3] Measurements from in situ measurement field campaigns and laboratory experiments have found that ice clouds are composed with nonspherical ice crystals with various habits [e.g., Heymsfield and Miloshevich, 2003; Bailey and Hallett, 2004]. Because ice cloud particles are small relative to the wavelengths of meteorological millimeter radar, the scattering properties of ice cloud particles can possibly be calculated using the Rayleigh scattering approximation [Schneider and Stephens, 1995]. The Rayleigh scattering approximation has been employed to calculate the scattering properties at low frequencies of below 35 GHz for ice crystals with habits of plate, column, and needle, which have been presented by oblate or prolate spheroidal models [e.g., Schneider and Stephens, 1995; Aydin and Walsh, 1999; Battaglia et al., 2001]. However, for the higher frequency of 94 GHz, the Rayleigh approximation generally does not produce accurate radar backscattering properties for these spheroidal models [Schneider and...
[4] To obtain accurate radar backscattering properties of nonspherical ice crystals, several numerical methods have been used to solve Maxwell’s equations for light scattering by nonspherical ice crystals [Kokhanovsky, 2006, 2007], for example, the T matrix [Mishchenko et al., 1996], the finite difference time domain (FDTD) [Yang and Liu, 1996a; Aydin and Tang, 1997; Sun et al., 1999], an improved geometrical optics method (IGOM) [Yang and Liu, 1996b], and the discrete dipole approximation (DDA) [Draine and Flatau, 1994]. Among these methods the FDTD and DDA methods have been used to calculate radar backscattering properties of nonspherical ice crystals since they are suitable for nonspherical ice crystals with complex habits at cloud radar frequencies [e.g., Schneider and Stephens, 1995; Liu and Illingworth, 1997; Aydin and Tang, 1997; Aydin and Walsh, 1999; Lemke and Quante, 1999; Okamoto, 2002; Sato and Okamoto, 2006].

[5] Most of the studying of the radar backscattering properties of nonspherical ice crystals are focused on columnar or planar crystals [e.g., Schneider and Stephens, 1995; Liu and Illingworth, 1997; Aydin and Tang, 1997; Okamoto, 2002]. In fact, ice clouds frequently consist of polycrystalline crystal forms composed of bullet rosettes or aggregates [e.g., Heymsfield and Miloshevich, 2003; Bailey and Hallett, 2004]. However, only a few works have investigated radar backscatter properties for these polycrystalline crystals. For example, Aydin and Walsh [1999] have investigated the radar backscattering properties at 35, 94, and 220 GHz for planar bullet rosettes with four branches and spatial bullet rosettes with six and eight branches using the FDTD method. Two types of stellar aggregates have been studied for radar backscatter characteristics at the frequencies of 35, 95, 140, and 220 GHz by Lemke and Quante [1999]. Recently, Sato and Okamoto [2006] have investigated the radar reflectivity factor and linear depolarization ratio at 95 GHz for bullet rosettes with five branches.

[6] The ice crystals with columnar or planar habits (hexagonal column, hollow, and plate), polycrystalline habits (bullet rosette and aggregate), and droxtal habits have been extensively used for retrieving microphysical and optical properties of ice clouds from satellite infrared and solar measurements [e.g., King et al., 2004, 2006; Yang et al., 2005; Baum et al., 2005a], such as the Moderate Resolution Imaging Spectroradiometer (MODIS) and Atmospheric Infrared Sounder aboard the Aqua satellite, which is a part of the satellite constellation A-Train. A-Train also includes CloudSat, the first satellite-based millimeter wavelength cloud radar at 94 GHz launched on 28 April 2006. A key aspect of the observing philosophy of the CloudSat mission is to combine the observations from the multiple sensors of the A-Train with the CloudSat observations [Stephens et al., 2002]. One of the important CloudSat science objectives is to evaluate cloud properties derived from other satellite sensors including those from the Aqua and operational sensors [Stephens et al., 2002; Buehler et al., 2007]. Using consistent ice crystal habits is critical for interpolating the combined measurements of cloud properties from multiple satellite sensors and for evaluating cloud properties. So it is necessary to develop the backscattering properties for CloudSat using the nonspherical ice crystal habits that are consistent with those extensively used for visible and infrared retrievals of ice cloud properties.

[7] The objective of this study is to investigate and parameterize the radar backscattering properties at 94 GHz for nonspherical ice crystals: hexagonal column, hollow, plate, bullet rosette, aggregate, and droxtal. In section 2 the DDA method used to compute the radar backscattering cross section is introduced. In section 3 the radar backscattering cross sections at 94 GHz from the DDA method are compared to those computed from the Lorenz-Mie theory, which uses spherical particles with equivalent maximum dimension, volume, projected area, and the ratio of volume to projected area for the six nonspherical ice crystals. The radar backscattering cross sections of the nonspherical ice crystals are parameterized as functions of maximum dimensions in section 4. In section 5 the parameterizations of radar backscattering cross sections are applied to ice clouds with a mixture of ice crystal habits and thereby to derive the relationships between the equivalent radar reflectivity factor and ice water content. The conclusion is presented in section 6.

2. Discrete Dipole Approximation Method

[8] Kokhanovsky [2006, 2007] have reviewed the various methods that are used to study the light scattering and absorption by nonspherical ice crystals. The T matrix method developed by Mishchenko et al. [1996] is efficient for spheroidal particles. The FDTD [Yang and Liu, 1996a; Sun et al., 1999] and IGOM [Yang and Liu, 1996b] methods have been extensively used to compute the single scattering properties of ice clouds at visible and infrared bands. The DDA method developed by Draine and Flatau [1994] has the greatest advantage of flexibility regarding ice crystals with arbitrary geometries. Its basic concept is the representation of a scatterer by an array of polarizable point dipoles on a cubic lattice with a lattice spacing d. The DDA method has been used to study the scattering by dusts and ice crystals with complicated shapes [e.g., Evans and Stephens, 1995; Kalashnikova and Sokolik, 2004; Liu, 2004; Kim, 2006; Sato and Okamoto, 2006; Hong, 2007]. In this study, dipole numbers N(Vid), where V is the volume of the scatter) are varied with particle sizes to ensure the criterion |m|kd < 0.5 (where m is the complex refraction index of the scatter, k = 2π/λ is the angular wave number, and λ is wavelength), which is required by the accuracy of the DDA computing (B. T. Draine and P. J. Flatau, User guide to the discrete dipole approximation code DDSCAT 6.1, 2004, available at http://arxiv.org/abs/astro-ph/0409262v2).

3. Backscattering Properties of Nonspherical Ice Crystals

[9] When an incident electric field is propagating toward a deterministic scatterer, the scattering properties of the scatterer is determined by the amplitude scattering matrix S

\[
\begin{bmatrix}
E_h' \\
E_v'
\end{bmatrix} = \begin{bmatrix}
\frac{e^{ikr}}{ikr} S_{hh} & S_{hv} \\
S_{vh} & S_{vv}
\end{bmatrix} \begin{bmatrix}
E_h \\
E_v
\end{bmatrix}
\]
where $E_h$ and $E_v$ describe the incident electric field and $E_h$ and $E_v$ describe the scattering electric field. The subscripts $h$ and $v$ denote the horizontal and vertical directions, respectively, and $r$ is the distance from the scatterer. The amplitude scattering matrix $S$ includes the copolar ($S_{hh}$ and $S_{vv}$) and cross-polar ($S_{hv}$ and $S_{vh}$) matrix elements. For a spherical particle the $S_{hv}$ and $S_{vh}$ are zero. The nonspherical ice crystals in an ice cloud, in general, have been assumed to be randomly orientated, so $S_{hh} = S_{vv}$ and $S_{hv} = S_{vh}$. On the basis of $S_{hh}$ ($S_{vv}$) the radar backscattering cross section can be determined. Additionally, with $S_{hv}$ ($S_{vh}$) the linear depolarization ratio can be further determined. Since CloudSat provides only information on radar reflectivity, the copolar matrix elements $S_{hh}$ ($S_{vv}$) are only considered in this study.

Six nonspherical ice crystals including hexagonal column, hollow, plate, six-branch bullet rosette, aggregate, and droxtal have been used for retrievals of optical and microphysical properties of ice clouds [e.g., King et al., 2004, 2006; Yang et al., 2005; Baum et al., 2005a]. The geometrical habits of the six nonspherical ice crystals are shown in Figure 1 and have been introduced in detail by Yang et al. [2005] and Hong [2007]. The ice crystal volumes and dipole sizes used for the DDA computing as a function of the maximum dimension $D$ are shown in Figure 2. Note that in this study $D$ refers to the maximum dimensions of original nonspherical particles. The pure ice density of 0.917 g cm$^{-3}$ is used for the six ice crystals. The refractive indexes of the ice crystals at the frequency of 94 GHz are from the data set [Warren, 1984] at the temperature of $-30^\circ$C. The $S_{hh}$ of nonspherical ice crystals with 46 maximum dimension $D$ in the range of $2 - 10,500 \mu$m are computed by the DDA model (B. T. Draine and P. J. Flatau, User guide to the discrete dipole approximation code DDSCAT 6.1, 2004, available at http://arxiv.org/abs/astro-ph/0409262v2) for which the geometric interface has been modified. The backscattering cross section $\sigma_{hh}$ is then determined by

$$\sigma_{hh} = \frac{4\pi}{k^2} |S_{hh}|^2.$$  \hspace{1cm} (2)

The scattering properties of nonspherical ice crystals have been computed by the Lorenz-Mie theory using spherical particles with equivalent maximum dimension $D$, volume $V$, projected area $A$, or ratio of volume to projected area ($V/A$) as those of nonspherical ice crystals [e.g., Lee et al., 2003; Donovan et al., 2004; Weinman and Kim, 2007]. In this study a similar method is used for calculating the $\sigma_{hh}$ using the Lorenz-Mie theory for the six nonspherical ice crystals. Moreover, the scattering properties of an individual nonspherical ice crystal are represented by those of a sphere with an adjusted effective bulk density in order to conserve the mass of the ice particle [Donovan et al., 2004]. Therefore the backscattering cross section $\sigma_{hh}$ in this study is adjusted by a factor of $\rho_2/\rho_1$ ($\rho_1$ is pure ice density and $\rho_2$ is the effective bulk density of the equivalent spherical ice particle). The factor of $\rho_2/\rho_1$ is derived by conserving the mass of the individual ice particle ($\rho_1 V = \rho_2 V_{equ}$), where $V_{equ}$ is the effective bulk density of the equivalent spherical particle. The factor of $\rho_2/\rho_1$ is then determined by

$$\rho_2/\rho_1 = \left( \frac{V_{equ}}{V} \right) \left( \frac{\rho_1}{\rho_2} \right) \left( \frac{A_{equ}}{A} \right)^{-1}.$$  \hspace{1cm} (3)

The backscattering cross section $\sigma_{hh}$ is then determined by

$$\sigma_{hh} = \frac{4\pi}{k^2} |S_{hh}|^2 \frac{\rho_2}{\rho_1} \frac{A_{equ}}{A}.$$  \hspace{1cm} (4)

Figure 1. Geometries of nonspherical ice crystals considered in this study.

Figure 2. (a) Ice crystal volume $V$ and (b) dipole size $d$ used for the DDA calculation as a function of maximum dimension $D$ for six nonspherical ice crystals, column, hollow, plate, rosette, aggregate, and droxtal. Note that the maximum dimension $D$ used in size coordinate refers to the maximum dimensions of original nonspherical particles.
\( \rho_e V_e \), where \( V_e \) is the volume of the equivalent sphere for the nonspherical ice crystal) using

For equivalent \( D \)

\[
\frac{\rho_e}{\rho_i} = \frac{6V}{\pi D^3},
\]

(3)

For equivalent \( V \)

\[
\frac{\rho_e}{\rho_i} = 1,
\]

(4)

For equivalent \( A \)

\[
\frac{\rho_e}{\rho_i} = \frac{3V\sqrt{\pi}}{4\sqrt{A^3}},
\]

(5)

For equivalent \( V/A \)

\[
\frac{\rho_e}{\rho_i} = \frac{16A^3}{9\pi V^2}.
\]

(6)

[12] Figure 3 shows the \( \sigma_{hh} \) for the six nonspherical ice crystals as a function of maximum dimension \( D \). Those calculated by the Lorenz-Mie theory using spherical particles with equivalent \( D \), \( V \), \( A \), and \( V/A \) are also shown in Figure 3 for comparison. The \( \sigma_{hh} \) are denoted by \( 10\log \sigma_{hh} \) in units of \( \text{dB mm}^{-2} \). The \( \sigma_{hh} \) increases with the increasing of \( D \) for all calculations. In general, the results from equivalent \( D \), \( V \), \( A \), and \( V/A \) show different features for column, hollow, plate, rosette, and aggregate for large particles with \( D \) over 1.0 mm. For droxtal the results from equivalent \( D \), \( V \), \( A \), and \( V/A \) are similar and close to those calculated from the DDA method. This is because the shape of droxtal is more spherical than others. For column, hollow, plate and rosette the \( \sigma_{hh} \) from equivalent \( V/A \) and \( D \) are distinctly different from those from the DDA method. Moreover, in general, equivalent \( V/A \) overestimates \( \sigma_{hh} \), while equivalent \( D \) underestimates \( \sigma_{hh} \). For column and hollow \( \sigma_{hh} \) from equivalent \( V \) are close to those from the DDA method for column and hollow with an exception at \( D \) is 7.0–9.0 mm. With respect to other representations, the \( \sigma_{hh} \) from equivalent \( V \) or \( A \) are closer to those from the DDA method for plate, while the \( \sigma_{hh} \) from equivalent \( V \) or \( A \) are closer to those from the DDA method for rosette. For aggregate the equivalent \( D \) can be used to represent the DDA calculations.

[13] Averaging the \( \sigma_{hh} \) over a particle size distribution, the mean backscattering cross section \( \overline{\sigma}_{hh} \) for the ice cloud is then derived. The \( \overline{\sigma}_{hh} \) for ice clouds consisting of nonspherical ice crystals are investigated. The \( \overline{\sigma}_{hh} \) is calculated over a particle size distribution using the following equation:

\[
\overline{\sigma}_{hh} = \frac{\int_{D_{\text{min}}}^{D_{\text{max}}} \sigma_{hh} N(D) dD}{\int_{D_{\text{min}}}^{D_{\text{max}}} N(D) dD},
\]

(7)

where \( N(D) \) is the number density of ice crystal particles with a \( D \). The \( D_{\text{min}} \) and \( D_{\text{max}} \) are the minimum and maximum sizes of \( D \), respectively, in the given particle size distribution.

[14] Gamma particle size distribution has generally been used for ice clouds. It has a style as \( \text{e.g., Baum et al., } 2005b \)

\[
N(D) = N_0 D^\mu e^{-\kappa D} = N_0 D^\mu e^{\frac{-\Delta \log D}{\Delta \log D}},
\]

(8)

where \( N_0 \) is the intercept, \( \kappa \) is the slope, \( \mu \) is the dispersion usually ranging from 0 to 2 and \( \mu = 2 \) is used for this averaging, and \( D_m \) is the median maximum dimension of the distribution. Different values of \( D_m \) less than 1.0 mm are used for particle size distributions. The mean value \( b = 2.2 \), which is derived by averaging \( b = 2.1 \) for the tropical ice clouds and \( b = 2.3 \) for the midlatitude ice clouds in field measurements \( \text{[Baum et al., } 2005b \)\] is used for the gamma particle size. The effective particle size \( D_e \) for the ice cloud with a given \( N(D) \) is calculated by \( \text{e.g., King et al., } 2004; \text{ Yang et al., } 2005; \text{ Baum et al., } 2005b \)

\[
D_e = \frac{3}{2} \int_{D_{\text{min}}}^{D_{\text{max}}} \frac{V(D)N(D)dD}{\int_{D_{\text{min}}}^{D_{\text{max}}} A(D)N(D)dD}.
\]

(9)

[15] The \( \overline{\sigma}_{hh} \) calculated from the DDA method for ice clouds consisting of different ice crystal habits as a function of \( D_e \) are shown in Figure 4. The \( \overline{\sigma}_{hh} \) calculated from the Lorenz-Mie theory using the equivalent \( D \), \( V \), \( A \), and \( V/A \) are also investigated for comparison. Note that the maxima of \( D_e \) for ice clouds is different for the six ice crystal habits because of their geometries and the limitation of \( D_m \) of 1.0 mm. The \( \overline{\sigma}_{hh} \) from equivalent \( V/A \) and \( A \) are almost the same as those from the DDA method for columns and hollow, while those from equivalent \( D \) pronouncedly underestimate \( \overline{\sigma}_{hh} \). For plate and rosette the \( \overline{\sigma}_{hh} \) of equivalent spheres and DDA method are the same when \( D_e \) are less than 80 \( \mu \). Moreover, the \( \overline{\sigma}_{hh} \) of equivalent \( V \) or \( A \) are closer to those from the DDA method for plate and rosette, respectively, with respect to other representations. The \( \overline{\sigma}_{hh} \) from equivalent \( D \) are also distinctly underestimated. For aggregate the \( \overline{\sigma}_{hh} \) from equivalent \( V/A \) and \( V \) are overestimated, and the \( \overline{\sigma}_{hh} \) from equivalent \( A \) are closer to those from the DDA method. Because of the strong sphericity, droxtal has the same \( \overline{\sigma}_{hh} \) from the DDA method and represented spheres.

4. Parameterization of Radar Backscattering Cross Sections

[16] The scattering properties of nonspherical ice particles are generally expensive in computing times and memories \( \text{[e.g., Lee et al., } 2003; \text{ Yang et al., } 2005; \text{ Kim, } 2006 \)\] In practice, parameterization schemes of the scattering properties of nonspherical ice crystals have been extensively used for solar, infrared, and microwave wavelengths \( \text{[e.g., Aydin and Walsh, } 1999; \text{ Yang et al., } 2000, 2005; \text{ Liu, } 2004; \text{ Kim, } 2006; \text{ Hong, } 2007 \)\] The scattering properties of nonspherical ice crystals are generally parameterized as functions of ice crystal sizes. In this study the \( \overline{\sigma}_{hh} \) computed from the
The DDA method are parameterized as functions of $D$ for the six ice crystal habits in the form of

$$\log \sigma_{hh} = \sum_{n=0}^{7} a_n \log^n D,$$  \hspace{1cm} (10)

where $\sigma_{hh}$ is in units of $\text{mm}^2$, $D$ is in units of $\text{mm}$, and $a_n$ is the fitting coefficients. The $a_n$ for the six ice crystal habits are shown in Table 1.

Figure 5a compares the $\sigma_{hh}$ from the parameterizations for the six nonspherical ice crystals with those from the DDA method. The results from the parameterizations are well in agreement with those from the DDA method.
particularly, for column, hollow, plate, and rosette. For aggregate and droxtal the feature of $\sigma_{hh}$ with a small reduction at 2.0–2.5 mm shown by the results from the DDA method does not occur for the results from the parameterizations.

[18] The $\bar{\sigma}_{hh}$ derived from the $\sigma_{hh}$ using the parameterizations and those calculated by the Lorenz-Mie theory using a sphere with equivalent $V$ are compared to those derived from the DDA method. A relative error is defined by the ratio of the difference between the $\bar{\sigma}_{hh}$ from the parameterizations (or the results from the Lorenz-Mie theory using a sphere with equivalent $V$) and those from the DDA method to those from the DDA method. The relative errors for ice clouds with the six ice crystal habits are shown in Figure 5b. The relative errors for those from the parameterizations for ice clouds consisting of column, hollow, plate, and rosette. For aggregate and droxtal the feature of $\sigma_{hh}$ with a small reduction at 2.0–2.5 mm shown by the results from the DDA method does not occur for the results from the parameterizations.

Figure 4. Mean radar backscattering cross sections at 94 GHz for ice clouds consisting of different habits ((a) column, (b) hollow, (c) plate, (d) rosette, (e) aggregate, and (f) droxtal) as a function of effective particle size $D_e$. The results calculated by the DDA method are compared to those from the Lorenz-Mie theory using the equivalent maximum dimension $D$, volume $V$, projected area $A$, and ratio ($V/A$) of volume to projected area.
Table 1. Fitting Coefficients for Parameterizing Backscattering Cross Sections as a Function of Maximum Dimension for Randomly Oriented Nonspherical Ice Crystals

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Column</th>
<th>Hollow</th>
<th>Plate</th>
<th>Rosette</th>
<th>Aggregate</th>
<th>Droxtal</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_0$</td>
<td>$-2.75947E+00$</td>
<td>$-2.88262E+00$</td>
<td>$-2.74664E+00$</td>
<td>$-3.33525E+00$</td>
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<td>$3.78938E+00$</td>
<td>$4.72372E+00$</td>
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</tr>
<tr>
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<td>$-1.86675E-01$</td>
<td>$-9.70183E-01$</td>
<td>$-9.78527E-01$</td>
<td>$-1.46326E+00$</td>
<td>$-1.15056E+00$</td>
</tr>
<tr>
<td>$a_3$</td>
<td>$-2.31878E-01$</td>
<td>$-2.02329E-01$</td>
<td>$2.09480E-01$</td>
<td>$4.44630E-01$</td>
<td>$1.57649E-01$</td>
<td>$1.35553E+00$</td>
</tr>
<tr>
<td>$a_4$</td>
<td>$-5.15793E-01$</td>
<td>$-5.41199E-01$</td>
<td>$6.17941E-01$</td>
<td>$6.24280E-01$</td>
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<td>$6.67469E-01$</td>
</tr>
<tr>
<td>$a_5$</td>
<td>$4.24583E-02$</td>
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<td>$1.16850E-01$</td>
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<td>$6.72013E-02$</td>
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<td>$a_6$</td>
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<td>$-7.95515E-02$</td>
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<tr>
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<td>$-2.64063E-02$</td>
<td>$-1.89979E-02$</td>
<td>$-1.13789E-01$</td>
</tr>
</tbody>
</table>

5. Application of Parameterizations for Ice Clouds

[19] The mixture of ice crystal habits has been found in ice clouds [e.g., Heymsfield et al., 2002; King et al., 2004, 2006; Yang et al., 2005; Baum et al., 2005a]. Considering the habit distribution $f(D)$ in an ice cloud, the $\sigma_{hh}$ in equation (7) is then calculated by

$$\sigma_{hh}(D) = \frac{\int_{D_{min}}^{D_{max}} \left[ \sum_{i=1}^{N} f_i(D) \sigma_{i,hh}(D) \right] N(D)dD}{\int_{D_{min}}^{D_{max}} N(D)dD},$$

where $\sum_{i=1}^{N} f_i(D) = 1$, $i$ denotes the ice crystal habit in the ice cloud, and the $\sigma_{i,hh}$ is $\sigma_{hh}$ for the ice crystal habit $i$. Different ice crystal habit distributions have been used for ice cloud retrievals from solar and infrared measurements [e.g., Yang et al., 2005; Baum et al., 2005b; King et al., 2004, 2006]. Recently, Baum et al. [2005a] derived a new ice cloud habit distribution from the measurements of airborne sampling probes and balloon-borne replicators in several field experiments. This distribution has been used for current ice cloud retrievals from the MODIS data [King et al., 2006]. It consists of 100% droxtals when $D < 60 \mu m$; 15% bullet rosettes, 50% solid columns, and 35% plates when $60 < D < 1000 \mu m$; 45% hollow columns, 45% solid columns, and 10% aggregates when $1000 < D < 2500 \mu m$; and 97% bullet rosettes and 3% aggregates when $D > 2500 \mu m$.

[20] Radar has an advantage of measuring the vertical structures of ice clouds. Combining the measurements from radar, solar, and infrared sensors, it has an opportunity to obtain the ensemble information of an ice cloud system [e.g., Stephens et al., 2002; Buehler et al., 2007]. A basis for using the measurements from multiple sensors is consistent microphysical properties of ice clouds for their retrievals. To be consistent with the ice crystal habit distribution used by solar and infrared measurements, the $\sigma_{hh}$ is investigated for hollow, plate, rosette, and aggregate are in the range of $-3 \pm 2\%$, which are much smaller than those from the calculations using the equivalent $V$. For ice clouds consisting of droxtal the $\sigma_{hh}$ from the DDA method are well represented by those from the equivalent $V$. However, the relative errors for the parameterizations are still in the range of $-2 \pm 2\%$ when the $D_e$ are less than 350 $\mu m$.

Figure 5. (a) Parameterizations of radar backscattering cross sections at 94 GHz for six nonspherical ice crystals (column, hollow, plate, rosette, aggregate, and droxtal) as a function of maximum dimension $D$. (b) Relative errors of mean radar backscattering cross sections for ice clouds consisting of different habits as a function of effective particle size $D_e$. The relative errors are for results from parameterizations and those calculations using the equivalent $V$ with respect to those calculated by the DDA method.
ice clouds with the crystal habit distribution for the MODIS ice cloud retrieval in this study. The $D_e$ in equation (9) for the ice cloud with a given $N(D)$ and $f(D)$, which is the same as used by Baum et al. [2005a], is then reformed as [e.g., King et al., 2004; Yang et al., 2005; Baum et al., 2005b]

$$D_e = \frac{1}{2} \int_{D_{\text{min}}}^{D_{\text{max}}} \left[ \sum_{i=1}^{N_e} f_i(D) V_i(D) \right] N(D) dD. \quad (12)$$

[21] The $\tau_{hh}$ from the DDA method, parameterizations, and equivalent $V$ for the ice cloud are shown in Figure 6. The relative errors for the parameterizations and the equivalent $V$ with respect to those calculated by the DDA method.

$$Z_e = \frac{\lambda^4}{\pi^3|K|} \int_{D_{\text{min}}}^{D_{\text{max}}} \left[ \sum_{i=1}^{N_e} f_i(D) \sigma_{i,\text{hh}}(D) \right] N(D) dD. \quad (13)$$

for ice clouds, where $|K|$ is the dielectric factor for water and approximately 0.93 [e.g., Atlas et al., 1995].

[23] The gamma distribution in equation (8) is used to derive the $Z_e$-IWC for ice clouds. The IWC is $1.17$ with a limitation of less than 1.0 mm. The parameterizations of $\sigma_{hh}$ and calculated $\sigma_{hh}$ from the DDA method are used for the calculation of $Z_e$. The coefficients $a$ and $b$ of $Z_e$-IWC for clouds consisting column, hollow, plate, rosette, aggregate, and droxtal are in Table 2. The coefficients $a$ and $b$ derived from the parameterizations and DDA method are close. The normalized biases (NB) and normalized standard errors (NSE) defined by Aydin and Tang [1997] in estimating IWC using $Z_e$-IWC are also provided in Table 2. The NB and NSE are larger for IWC > 200 mg m$^{-3}$ with respect to those for IWC < 200 mg m$^{-3}$. For ice clouds with column, the parameterizations have absolute values of less than 3%. This indicates that the parameterizations of radar $\sigma_{hh}$ are efficient for calculating the $\tau_{hh}$ for ice clouds with the ice crystal habit distribution.

$$Z_e = Z_i \frac{|K|^2}{|K_e|^2} \quad (14)$$

Table 2. Coefficients $a$ and $b$ for the Relationships Between IWC and $Z_e$ for Ice Clouds Consisting of Nonspherical Ice Crystals$^a$

<table>
<thead>
<tr>
<th>Habit</th>
<th>DDA Calculation</th>
<th>IWC &gt; 0.20 g m$^{-3}$</th>
<th>IWC &lt; 0.20 g m$^{-3}$</th>
<th>Parameterization</th>
<th>IWC &gt; 0.20 g m$^{-3}$</th>
<th>IWC &lt; 0.20 g m$^{-3}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$a$</td>
<td>$b$</td>
<td>NB, %</td>
<td>NSE, %</td>
<td>$a$</td>
<td>$b$</td>
</tr>
<tr>
<td>Column</td>
<td>0.0781</td>
<td>0.4876</td>
<td>2.9</td>
<td>11.1</td>
<td>0.5</td>
<td>7.9</td>
</tr>
<tr>
<td>Hollow</td>
<td>0.0819</td>
<td>0.4811</td>
<td>3.2</td>
<td>11.3</td>
<td>0.5</td>
<td>8.4</td>
</tr>
<tr>
<td>Plate</td>
<td>0.0695</td>
<td>0.4271</td>
<td>-0.3</td>
<td>8.5</td>
<td>0.1</td>
<td>2.5</td>
</tr>
<tr>
<td>Rosette</td>
<td>0.1024</td>
<td>0.4545</td>
<td>-2.3</td>
<td>7.6</td>
<td>-0.1</td>
<td>4.3</td>
</tr>
<tr>
<td>Aggregate</td>
<td>0.0600</td>
<td>0.4044</td>
<td>-35.0</td>
<td>34.7</td>
<td>1.8</td>
<td>9.4</td>
</tr>
<tr>
<td>Droxtal</td>
<td>0.0315</td>
<td>0.4168</td>
<td>-36.5</td>
<td>32.4</td>
<td>0.9</td>
<td>17.1</td>
</tr>
<tr>
<td>Mixing</td>
<td>0.0769</td>
<td>0.4720</td>
<td>-13.4</td>
<td>17.6</td>
<td>1.5</td>
<td>7.4</td>
</tr>
</tbody>
</table>

$^a$Nonspherical ice crystals: IWC = $aZ_e^b$, where IWC is g m$^{-3}$ and $Z_e$ is mm$^6$ m$^{-3}$. The coefficients are derived from the DDA calculated scattering properties and parameterizations. The NB and NSE used in estimating IWC are also given.
and below 9% for IWC \(<\) and Heymsfield, from the equivalent sphere are \(Z\) when \(D\) Brown et al. \(Z\)s are applied to derive the \(Z\). Schneider and Stephens for ice clouds with an ice over \(D\) from equivalent spheres and the DDA \(Z\) (mm \(V\)s from parameterizations and those from the \(V\)s Comparison of the relationships between \(Z\) for the six nonspherical ice crystals on the \(Z\) (Bailey and Hallett \(V\)s from parameterizations and those from the \(D\) and ice \(V\)s, 2004\]. The \(Z\) from the DDA \(V\)s from the DDA method, \(m\) [2005a]. The \(Z\) from the DDA method (Table 1). The \(Z\) of the six nonspherical \(Z\)s provide an efficient way to derive the \(Z\) from the \(Z\)s derived from the \(Z\)s, which are much larger than those for IWC < 0.2 g m\(^{-3}\). The ice clouds with a mixture of habits have absolute values of NB and NSE of 11–18% for IWC > 0.2 g m\(^{-3}\) and below 9% for IWC < 0.2 g m\(^{-3}\), respectively.

[24] The \(Z\)-IWC relationship from the parameterizations for the ice cloud consisting of a mixture of ice crystal habits is compared to several previous \(Z\)-IWC relationships [Atlas et al., 1995; Brown et al., 1995; Schneider and Stephens, 1995; Aydin and Tang, 1997; Liu and Illingworth, 2000], which have been derived for ice clouds at 94 GHz from different ice cloud microphysical data on the basis of field experimental measurements or cloud model-generated data [Sassen et al., 2002]. Figure 7 shows the results for the comparison. These \(Z\)-IWC relationships show pronounced differences. This reveals that the \(Z\)-IWC relationship is strongly sensitive to the microphysical properties of ice clouds, such as the ice crystal habits, mixtures of ice crystal habits, and particle size distributions. Obviously, there is no universal \(Z\)-IWC relationship for all ice clouds [e.g., Atlas et al., 1995; Aydin and Tang, 1997; Liu and Illingworth, 2000; Sassen et al., 2002; Boudala et al., 2006]. So it is important to study the \(Z\)-IWC relationships for ice clouds with varying microphysical properties. The parameterizations of the radar backscattering properties for nonspherical ice crystals provide an efficient way to derive the \(Z\) for ice clouds and thereby to derive their \(Z\)-IWC relationships.

6. Conclusion

[25] Nonspherical ice crystals of various habits have been observed for ice clouds from in situ measurement field campaigns and laboratory experiments [e.g., Heymsfield and Miloshevich, 2003; Bailey and Hallet, 2004]. The remote sensing of ice cloud properties using millimeter wave radars requires accurately modeling the backscattering properties of ice clouds [e.g., Schneider and Stephens, 1995; Aydin and Tang, 1997; Sato and Okamoto, 2006]. The hexagonal column, hollow, plate, bullet rosette, aggregate, and droxtal (Figure 1), which have been used for passive infrared and visible [e.g., King et al., 2004, 2006; Yang et al., 2005; Baum et al., 2005a] and microwave [Hong, 2007] remote sensing techniques, are used to simulate radar backscattering properties at 94 GHz on the basis of the DDA method in this study.

[26] The backscattering cross sections \(\sigma_{hh}\) calculated by the DDA method for the ice crystals with the six habits are compared to those calculated by the Lorenz-Mie theory using spherical particles with equivalent maximum dimension \(D\), volume \(V\), projected area \(A\), and ratio of volume to projected area \(V/A\) (Figure 3). The \(\sigma_{hh}\) from the six nonspherical ice crystals, which are computed for 46 \(D\) ranging from 2 to 10,500 \(\mu m\), are sensitive to ice crystal habits and increase with the increasing of \(D\). The \(\sigma_{hh}\) from the DDA method are well represented by those calculated from the spherical particles with equivalent \(V\), \(D\), \(A\), and \(V/A\) when \(D\) is less than 1.0 mm. However, for particles with \(D\) over 1.0 mm, pronounced differences are found for the \(\sigma_{hh}\) calculated from equivalent spheres with respect to those from the DDA method. The \(\sigma_{hh}\) are averaged over gamma distributions with varying \(D\) less than 1.0 mm to obtain the mean backscattering cross sections \(\sigma_{hh}\) for ice clouds with the six ice crystal habits (Figure 4). In general, the \(\sigma_{hh}\) from the equivalent spheres represent those from the DDA method well for column, hollow, and droxtal. For plate, rosette, and aggregate the \(\sigma_{hh}\) from the equivalent sphere are almost the same as those from the DDA method only when effective particle sizes \(D_e\) are small. Moreover, their differences among \(\sigma_{hh}\) from equivalent spheres and the DDA method generally increase with increasing \(D_e\).

[27] The parameterizations of \(\sigma_{hh}\) are developed as a function of \(D\) for the six nonspherical ice crystals on the basis of the calculated \(\sigma_{hh}\) from the DDA method (Table 1). The parameterizations agree well with the calculations from the DDA method (Figure 5). The \(\sigma_{hh}\) derived from the parameterized \(\sigma_{hh}\), calculated \(\sigma_{hh}\) from the DDA method, and calculated \(\sigma_{hh}\) from the Lorenz-Mie theory using equivalent \(V\) spheres are compared. The differences between the \(\sigma_{hh}\) from parameterizations and those from the DDA method are much smaller than those between the Lorenz-Mie theory using equivalent \(V\) spheres and the DDA method with the exception of droxtal. The parameterizations of \(\sigma_{hh}\) are applied to derive the \(\sigma_{hh}\) for ice clouds with an ice crystal habit distribution from Baum et al. [2005a]. The results are compared to those from the DDA method and the Lorenz-Mie theory using the equivalent \(V\) spheres (Figure 6). The \(\sigma_{hh}\) from the Lorenz-Mie theory are strongly different from those from the DDA method, while those from the parameterizations are essentially the same as those from the DDA method.

[28] On the basis of the calculated \(\sigma_{hh}\) from the parameterizations and the DDA calculations the relationships between the equivalent radar reflectivity factor \(Z\) and ice water content IWC for ice clouds consisting of the six ice crystal habits and an ice crystal habit distribution are...
derived using the gamma distributions with the parameters from Heymsfield et al. [2002] and Heymsfield [2003] (Table 2). The derived coefficients for the $Z_{	ext{ZWC}}$ relationships on the basis of the parameterizations and the DDA calculations are very close. Ice clouds consisting of column, hollow, plate, rosette, and a mixture of ice crystal habits have much smaller fitting errors for the $Z_{	ext{ZWC}}$ relationships with respect to those for aggregate and droxtal. For all ice clouds, the fitting errors are much smaller for ice clouds with IWC $< 0.2$ g m$^{-3}$ than those with IWC $> 0.2$ g m$^{-3}$. The $Z_{	ext{ZWC}}$ relationship for an ice cloud with a mixture of ice cloud habits is compared to several previous $Z_{	ext{ZWC}}$ relationships derived from modeling or field measurements for ice clouds (Figure 7). The pronounced differences among these relationships confirm the sensitivity of $Z_{	ext{ZWC}}$ relationships to microphysical properties of ice clouds including ice crystal habits, mixtures of ice crystal habits, and particle size distributions.

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