ABSTRACT
Experimental investigation of the sliding-velocity dependence of friction in rock is necessary to understand slip instability and earthquake phenomena, but a capability to conduct sliding friction experiments at high normal loads and at earthquake slip rates (1 m/s) must be developed. Velocity-dependent friction is ideally studied by imposing step-wise changes in rate of sliding. Velocity steps to earthquake slip rates must occur in a few milliseconds and requires high accelerations, so feedback control is not applicable because the rise time of actuators that can be used for our application is large relative to the rise time of the required velocity response. For this reason, we analyze the capability of a high-speed loading system with high load capacity to impose a velocity step, followed by constant rate of sliding, using only several preset passive-control variables. Assuming representative transient-friction response of rock test-samples, preset passive-control variables of the loading system may be obtained such that the deviation from the ideal velocity-step is minimized. A parametric exploration of the preset passive-control variables and test-sample behavior is used to define the machine workspace, and to optimize the key design characteristics of the loading system, such that the desired load paths for a range of expected sample behaviors may be achieved.

KEYWORDS
Experimental rock deformation, high-speed loading system, dynamic analysis, workspace analysis.

INTRODUCTION
Defining velocity-dependent friction constitutive relations is necessary to understand slip instability and earthquake phenomena. One characteristic that has a significant importance to the stability of frictional systems is the transient weakening and strengthening of friction associated with changes in the sliding rate (Dieterich, 1978; Ruina, 1983). The rate and state-variable constitutive relations that have been formulated are consistent with experimental observations and can successfully describe the effect of sliding rate, temperature and high normal stress on frictional strength of rocks. However, these constitutive relations are based only on quasi-static experiments; laboratory testing apparatuses typically operate at low sliding rates, i.e., < 1 mm/s (Linker & Dieterich, 1992; Chester, 1994; Marone, 1998). It is not clear whether these constitutive relations can describe behaviors at sliding rates during an earthquake, which can be on the order of 1 m/s (e.g., Niemeijer et al., 2012).

The rate and state friction constitutive relations are best explored through rate-stepping tests. Step-wise changes in sliding are employed to investigate dependence of friction on sliding velocity (e.g., Marone, 1998). Sliding rates greater than 0.01 m/s can generate significant increase in temperature at high normal loads due to frictional heating. A high rate of work and resultant increase in temperature may activate a number of weakening mechanisms, such as generation of pressurized fluids or melt. Pressurized pore fluids and melt along the sliding surface can act like a lubricant and decreases the frictional strength of rock surfaces (Di Toro et al. 2006). In this case the coefficient of friction can undergo a fivefold decrease to values as low as 0.1. Such a decrease in coefficient of friction after onset of sliding at seismic rates is referred to as dynamic weakening, and the decrease is usually modeled with an idealized exponential decay.

Recent advances in laboratory instrumentation have led to construction of high-speed rotary shear apparatuses and promoted significant advances in earthquake simulation. Due to
technical and instrumentation limitations, there is limited experimental capability to investigate rock friction at high normal stress, sliding rate and acceleration conditions that simulate those of the earthquake source (e.g., Chang et al., 2012). An additional advance in friction testing capability may be possible by incorporating a high-load, high-speed actuator in a biaxial apparatus employing a double-direct shear sample configuration for testing friction (Figure 1). In a double direct shear test configuration, three blocks of rock are loaded in a direction perpendicular to the contacting surfaces by a normal stress \( \sigma_n \). The two lateral blocks are stationary with respect to one another, and the central block is loaded with a force to generate shear stress on the contacting surfaces to cause sliding between the central and lateral blocks. Biaxial apparatuses are used broadly for conducting double-direct shear tests at low sliding rates (e.g., Dieterich, 1978; Mair & Marone, 1999; Collettini et al., 2009; Carpenter et al., 2009; Noda & Shimamoto, 2010). Relatively large samples can be used and displacements can be large enough to investigate frictional properties of rock. Since these experiments are done at very slow sliding rates, servo-hydraulic systems are ideal loading systems.

Figure 1. Double direct shear configuration; \( \sigma_n \) is the normal stress and \( \sigma_s \) is the axial stress that generates the shear stress along the contacting surfaces and drives frictional sliding.

Herein, we consider incorporating a high-speed pneumatic actuator and a hydraulic damper of an existing triaxial apparatus into a biaxial load frame in order to perform double-direct shear tests at high-speed conditions. The existing high-speed actuator was used to conduct fracture experiments at intermediate strain rates (Green et al. 1968; Logan & Handin, 1971); however, this loading system was not designed or used to achieve the intended velocity-step pattern we require for friction experiments. For conducting velocity-step experiments at seismic sliding rates (1 m/s), the loading system needs to achieve high acceleration rates with very small rise time during steps (about 2 milli-seconds) and maintain constant velocity for about 2-4 cm without requiring feedback control. Feedback control is not applicable for our purpose because the rise time of an actuator that is actively controlled is large relative to the rise time of the velocity response that is desired in these experiments. In other words, if we use any actively controlled device in our high-speed loading system, the device will not respond fast enough to control the velocity response. For this reason, we can only adjust some Preset Passive-Control Variables (PPCV) before each experiment to achieve the intended response. The PPCV do not change during an experiment. Three PPCV that we use in this study are: 1. Initial pressure in the pneumatic cylinder \( (P_{\alpha 1}) \), 2. The area of the orifice at the exhaust of the pneumatic cylinder \( (A^*) \), 3. Damping coefficient which is set by adjusting the needle valve at the orifice of the damper \( (C_d) \) (Figure 2). Achieving velocity-steps for high-speed friction experiments is quite challenging because samples may dramatically weaken during the experiment. However, it will be shown in this paper that the desired load path can be generated for the range of expected sample behaviors by modifying the design of the existing high-speed loading system and presetting the PPCV to appropriate values before each operation.

In the following sections, the structure of the loading system is presented, the model formulation is derived and the effects of PPCV on the system response are studied. Next, PPCV are obtained in such a way that the desired velocity-step is achieved. Finally, the workspace of the high-speed loading system is obtained and the optimum design requirements for the expected range of sample behaviors are determined.

**Description and Operation of the Proposed Apparatus**

The biaxial apparatus consists of a stiff, stationary load frame with a vertical and a horizontal-axis loading capability (Figure 2). The horizontal-axis hydraulic cylinder is not shown in Figure 2 to avoid complexity. This hydraulic cylinder operates at slow rates under closed-loop servo-control in displacement and load feedback, and generates the normal stress \( \sigma_n \) in the double-direct shear sample configuration. The pneumatic cylinder is double acting and is affixed to the frame. The rod of the pneumatic cylinder is connected to a double-acting hydraulic cylinder that acts as a damper to limit the velocity of the pneumatic piston.

![Figure 2. Schematic drawing of the apparatus; the horizontal hydraulic cylinder is not shown](image-url)
The pneumatic cylinder is fired in a single stroke, quick blow-down mode to impart high-velocity displacement to the sample assembly. Gas pressures up to 20 MPa are used to achieve about 1.5 MN load capability by a 32 cm diameter piston. Both chambers are filled with pressurized helium, which has low viscosity and density properties to achieve quick blow-down rates. The pneumatic cylinder is fired by opening a large-diameter exhaust port of the lower gas chamber using a quick acting valve (QAV). The reduction in gas pressure in the lower chamber creates a net force on the pneumatic cylinder plate that accelerates and displaces the cylinder rod downwards. The exhaust port employs adjustable orifice sizes to pre-set the mass flow rate of the exhausting gas.

The two chambers of the hydraulic damper are connected through an adjustable orifice that can be set by a micrometer screw and a needle valve to change the damping behavior. The duration of the high-speed operation is very short (a few milliseconds), which does not allow control of either the exhaust or damping orifice size during blow-down; however, by presetting the size of the exhaust and damping orifices, and by using different starting gas pressures, significantly different loading rates can be achieved by the pneumatic loading system.

Using the loading system in a biaxial load-frame, a double-direct shear friction experiment is conducted by activating the horizontal-axis hydraulic cylinder under load-feedback servocontrol to establish the normal stress across the sliding surfaces of the three block sample assembly. At this stage equal gas pressure is established in the two chambers of the pneumatic cylinder. The frictional sliding experiment begins by activating the QAV. As the vertical load by the pneumatic cylinder increases, the pneumatic rod pushes against the sample. Because the helium gas is released from the lower chamber of the cylinder so quickly, and the mass of the pneumatic rod and load column components is relatively small, accelerations approaching 100g are possible. As the pneumatic rod, load column and central block of the sample assembly approach high speeds, the resistive force of the damping cylinder is increased and acts to arrest the acceleration.

An important operational constraint of the high-speed pneumatic loading system is that because there is no real-time control, the velocities achieved during operation of the pneumatic cylinder depend on the adjustable parameters of the system (damping orifice, exhaust orifice and initial gas pressure) and the frictional behavior of the sample. Generally the frictional behavior is unknown (defining the constitutive behavior is the goal of the experiments). Using an accurate model of the loading system behavior, and approximate knowledge of the sample behavior, the PPCV of the pneumatic cylinder can be pre-set to accelerate and decelerate the piston and to achieve the target velocity.

Model Formulation

The dynamics of the pneumatic cylinder is investigated in two stages. Stage 1 is initiated by opening the QAV and the gas starts to exhaust through the QAV. During this stage differential pressure generates a driving force. As long as the driving force is less than the initial resisting force of the sample \( F_{\text{o}} \), the piston and load column do not move. Once the driving force of the pneumatic rod exceeds \( F_{\text{o}} \), the rod, load column, and central block of the sample assembly accelerate downward, initiating stage 2. Stage 2 continues until the loading column or the piston plate in the pneumatic cylinder hits a mechanical stop.

Stage 1: Pneumatic loading before piston starts to move

In this stage, the piston is not moving, and consequently the gas pressure is constant in the upper chamber. So, we can model the lower chamber as a pressurized tank with constant volume venting out gas through an orifice (Figure 3). Different parameters are defined in Table 1. Initial conditions such as the initial pressure and temperature of the gas are known, and by using the law of conservation of mass, pressure can be determined as a function of time. For this purpose, one may write (Cengel & Cimbala, 2010),

\[
\frac{d}{dt} \int_{CV} \rho dV + \int_{CS} \rho (v, n) dA = 0
\]

(1)

Here, \( CV \) (Control Volume) and \( CS \) (Control Surface) are the volume and surface of the lower chamber respectively. This relation states that the rate of change in mass of the gas inside the \( CV \) is equal to the mass flow rate passing surface boundary \( CS \). The volume of the lower chamber \( (V_{02}) \) is constant during Stage 1 because the piston is not moving, and only the density of gas changes because gas is exhausting the lower chamber. So we may write,

\[
\frac{d}{dt} \int_{V_{02}} \rho dV = \frac{d \rho}{dt} V_{02}
\]

(2)

Where,

\[
V_{02} = A_p h_2
\]

(3)

Also, the net mass flow rate passing the \( CS \) can be obtained as the mass flow rate of the exhausting gas through the orifice,

\[
\int_{CS} \rho (v, n) dA = \rho \dot{v} A^* = \rho \dot{A} A^*
\]

(4)

By substituting equations (2-4) in Eq. (1), we have,

\[
\frac{d \rho}{dt} A_p h_3 + \rho \dot{\rho} A^* = 0
\]

(5)

The downstream pressure is much lower than the pressure inside the tank so the gas flow through the orifice will be choked. This means that Mach number \( (M = v_{\text{fluid}} / v_{\text{sound}}) \) is equal to 1 at the throat of the orifice and gas passes the throat at the speed of sound. This holds when the pressure ratio of the downstream to upstream is lower than \( (2/\gamma + 1)^{\gamma / \gamma - 1} \) (Rathakrishnan, 2010). This ratio is equal to 0.48 for helium, so the gas flow is choked as long as the pressure in the tank is equal or more than about twice the downstream pressure. One can obtain the speed of sound as \( v_{\text{sound}} = \sqrt{\gamma RT} \) (Rathakrishnan, 2010). At the throat of the orifice \( M = 1 \), so the
speed of exhausting gas is equal to speed of sound at the throat,
\[ v^* = \sqrt{\frac{\gamma RT^*}{\rho}} \]  
(6)

We can assume that there is no heat transfer during the operation because it happens very fast. So, isentropic flow relations for a converging-diverging nozzle (i.e. flow through the orifice inside the exhaust port) can be used to relate temperature, density and pressure as follows (Rathakrishnan, 2010).
\[ T^* = \frac{T}{1 + \frac{\gamma - 1}{2} M^2} \]  
(7)

Where \( M = 1 \),
\[ \rho^* = \rho \left( \frac{2}{\gamma + 1} \right)^{1/\gamma} \]  
(8)

\[ T^* = T_{i0} \left( \frac{\rho}{\rho_{i0}} \right)^{\gamma - 1} \]  
(9)

Substituting equations (6-9) into Eq. 5, we have,
\[ \frac{A_p h^2}{2} \frac{d\rho}{dt} + A^* \sqrt{\frac{\gamma RT_{i0}}{(\rho_{i0})^{\gamma - 1}} \left( \frac{2}{\gamma + 1} \right)^{\frac{\gamma^2}{2}}} \rho^* \frac{\gamma^2}{2} = 0 \]  
(10)

**Stage 2: Pneumatic loading after pneumatic piston moves**

In this stage, the piston starts to move and volume of the lower chamber is no longer constant,

\[ V_2 = A_p (h_2 - y) \]  
(11)

The ideal gas law can be written as,
\[ P_2 V_2 = m_g RT_2 \]  
(12)

Temperature can be obtained as a function of pressure based on isentropic flow relations (Rathakrishnan, 2010),
\[ T_2 = T_{i0} \left( \frac{P_2}{P_{i0}} \right)^{\gamma - 1} \]  
(13)

Substituting Eq. (13) in Eq. (12) we may write,
\[ m_g = \frac{P_2^{\gamma - 1}}{RT_2} P_{i0}^{-\frac{1}{\gamma}} V_2 \]  
(14)

where, \( P_{i0}, T_{i0}, \rho_{i0} \) are constant at Stage 2. So, by differentiating Eq. (14) with respect to time we get,
\[ \dot{m}_g = \frac{P_{i0}^{\gamma - 1}}{RT_2} \left( \frac{V_2}{\gamma} \right) P_2^{-\frac{1}{\gamma}} \frac{dP_2}{dt} + \frac{1}{\gamma} \left( \frac{V_2}{\gamma} \right) P_2^{-\frac{1}{\gamma}} \frac{dV_2}{dt} \]  
(15)

On the other hand, the rate of change in mass inside \( V_2 \) is equal to mass flow rate of the exhausting gas at the orifice throat.
\[
\dot{m}_g = -\rho^* v^* A^*
\]  
(16)

Again based on isentropic flow relations we can write (Rathakrishnan, 2010),

\[
\rho^* = \rho_2 \left( \frac{2}{\gamma+1} \right)^{\frac{1}{\gamma-1}}
\]
(17)

\[
\rho_2 = \rho_{\text{ef}} \left( \frac{P_2}{P_{\text{ef}}} \right)^\gamma
\]
(18)

And,

\[
T^* = \frac{2}{\gamma+1} T_2
\]
(19)

\[
T_2 = T_{\text{ef}} \left( \frac{P_2}{P_{\text{ef}}} \right)^{\frac{1}{\gamma}}
\]
(20)

Substituting equations (6) and (17-20) into Eq. (16),

\[
\dot{m}_g = -A^* \rho_{\text{ef}} \sqrt{\frac{T_{\text{ef}}}{\gamma}} \left( \frac{2}{\gamma+1} \right)^{\frac{1}{\gamma-1}} \left( \frac{P_2}{P_{\text{ef}}} \right)^{\frac{1}{\gamma}}
\]
(21)

Now by equating equations (15) and (21) one can write,

\[
\frac{P_{\text{ef}}}{RT_{\text{ef}}} A_p \left( \frac{h_2 - y}{\gamma} \right) \frac{dy}{dt} \frac{P_2}{P_{\text{ef}}} - \frac{1}{P_2} \frac{dP_2}{dt} + \dot{m}_g = 0
\]
(22)

Also, by using Newton’s law the equation of motion can be written as,

\[
(P_1 - P_2)A_p - F_s = m\ddot{y} + c_d \dot{y}^2
\]
(23)

The damping force is proportional to the pressure difference between the two chambers of the damper which is mainly due to pressure drop at the adjustable orifice of the damper. Based on Bernoulli’s principle (Cengel & Cimbala, 2010), the flow pressure drop is proportional to flow velocity squared, and flow velocity is proportional to piston’s velocity due to conservation of mass of fluid inside the chambers of the damper. As a result, the damping force is modeled by \(c_d \dot{y}^2\) in Eq. (23). Magnitude of \(c_d\) can be set by adjusting a needle valve at the damper’s orifice. Also, based on isentropic expansion relations (Rathakrishnan, 2010), one can obtain pressure in the upper chamber of the pneumatic cylinder (\(P_1\)) during Stage 2,

\[
P_1 = P_{\text{ef}} \left( \frac{V_{\text{ef}}}{V_1} \right)^\gamma
\]
(24)

since the summation of volumes of lower and upper chambers of the pneumatic cylinder is constant, one can obtain \(V_1\) as,

\[
V_1 = V_{\text{ef}} + V_{\text{ef}} - V_2
\]
(25)

Substituting equations (24-25) and Eq. (11) in Eq. (23), we have,

\[
\left( P_{\text{ef}} \left( \frac{V_{\text{ef}}}{V_1} \right)^\gamma \right) A_p - F_s = m\ddot{y} + c_d \dot{y}^2
\]
(26)

Equations (21-22) and (26) are the governing equations of the high-speed loading system, and all of the terms in these equations are written as a function of \(P_2, \ y\) and their time derivatives. The elasticity of gas inside the pneumatic cylinder is included in these equations as we used the relations between volume and pressure in deriving the governing equations (e.g. Eq. (24)) which relates displacement of the piston to a change of pressure in the chambers and accordingly the force acting on the piston. So, the elasticity of gas does not need to be considered separately. For instance, consider a case when the piston is moved downward from the equilibrium position by an external disturbance. In this case, \(V_1\) increases and \(V_2\) decreases and consequently \(P_2\) increases and \(P_1\) drops. The change in pressure of chambers generates a force upward which tends to move piston up. This can lead to vibrations of the piston due to elasticity of the gas. Such vibrations can be observed in the response of the presented system when damping coefficient is low and \(F_s\) drops sharply (which acts like an un-damped disturbance). In this study, appropriate values of coefficient of damping have been employed to avoid such vibrations. Also, all rods in the load column are designed to have a relatively high stiffness. So, they can be considered as rigid bodies because deflections of rods are very small and relative motion of two ends of each rod is negligible relative to translational motion of the whole rod.

**Dynamic analysis of the loading system**

As mentioned in previous sections, the friction force drop during dynamic weakening in a high speed experiment can be modeled as an exponential decay. Accordingly, the behavior of the sample can be defined by using three parameters: 1. Initial friction force (\(F_{\text{f0}}\)), 2. Magnitude of the force drop during dynamic weakening (\(\Delta F_s\)), 3. A characteristic displacement called \(D_c\), in which 90% of the force drop occurs. These parameters are illustrated in Figure 4. To achieve the desired velocity-step the three PPCV (\(P_{\text{ef}}, A^*, c_d\)) are adjusted before each experiment.

![Figure 4. Friction force drop modeled with an exponential decay using \(F_{\text{f0}}, \Delta F_s\), and \(D_c\)](image-url)

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Figure 5 illustrates the response of the pneumatic piston and damper loading system for a sample with $F_{o}=200$ kN, $D_c=5$ mm and $\Delta F_o/F_{o}=10\%$. To achieve the desired target velocity-step (abrupt step to high velocity and then sliding at constant velocity), the initial pressure is set to 12 MPa, $A_1=2.32$ cm$^2$, and $c_d=52$ kN.s$^2$.m$^{-2}$. The forces in the system during stages 1 and 2 of the loading are depicted in Figure 5, specifically the force exerted to piston by the gas in the upper chamber ($F_{upper}$) and the lower chamber of the pneumatic cylinder ($F_{lower}$) and the force exerted by the damper are shown as a function of time after the QAV is opened. At $t=0$, the QAV operates and gas exhausts from the lower chamber. As a result, the pressure in the lower side drops. However, the piston does not move until the difference in the pressure of the upper and lower chamber is large enough to exceed the sample force ($F_{upper} - F_{lower} > F_{o}$). For this case stage 2 is started after 4 milli-seconds. Throughout stage 1 the pressure in the upper chamber remains constant. The maximum damping force during the 2 cm stroke is 61 kN.

![Figure 5](image)

**Figure 5.** Forces exerted to the piston from the lower chamber, upper chamber and the damper; during Stage 1, pressure in the upper chamber is constant and the pneumatic piston is not moving, during stage 2 the gas pressure in the both chambers decreases.

The force generated by the loading system is $F_{upper} - F_{lower} - F_{damping}$ and if we subtract $F_o$ from this force we get the net force that accelerated the piston. In other words,

$$F_{net} = A_p (P_1 - P_2) - F_{damping} - F_o$$  \hspace{1cm} (27)

The net force has a pulse-like characteristic, it jumps to the maximum value, then drops to zero and stays zero for the rest of the stroke (Figure 6), otherwise the desired velocity-step cannot be achieve. As shown in Figure 7 velocity response follows the desired target path for this case. 1 m/s velocity is achieved in less than 2 mm of displacement and it stays almost constant for 2 cm. In the next section, we present a parametric study of loading system response for different values of system parameters.

![Figure 6](image)

**Figure 6.** Net force accelerating the moving parts ($F_{upper} - F_{lower} - F_{damping} - F_o$)

![Figure 7](image)

**Figure 7.** Velocity of the piston as a function of displacement for $F_o=200$ kN, $D_c=5$ mm and $\Delta F_o/F_{o}=10\%$; the target velocity (1 m/s) is achieved in less than 2 mm

### Effects of different parameters on the response

In previous section the response of the loading system for a specific case was presented. The response could follow the desired pattern with a very small error. However, it’s not always possible to get the desired response. The ability to generate the desired velocity-step depends on both sample behavior and loading system characteristics. To explore this ability, in this section, effects of different parameters on the system behavior are investigated.

Figure 8 illustrates the response for three different values of $F_o$ (100 kN, 400 kN, 700 kN) while $\Delta F$s is set to zero so that we explore the effect of the $F_o$ more clearly. In all cases, the initial pressure is equal to 15 MPa and $c_d=52$ kN.s$^2$.m$^{-2}$. An ideal velocity curve can be achieved when $F_o=400$ kN. However, velocity drops enormously for $F_o=700$ kN. As the piston moves downward, the pressure decreases in the upper chamber of the pneumatic cylinder due to the volume expansion. For this reason, in this case ($F_o=700$ kN), the driving force of the loading system gets smaller than the resistive forces after 5 mm of displacement, and pneumatic piston decelerates. Moreover, in this case, a large orifice size is necessary to get to 1 m/s for those $P_0$ and $F_o$, otherwise the maximum velocity will be too low. On the other hand, if the initial pressure is too high relative to $F_o$, velocity increases slightly and deviates from the ideal curve. This can be seen in Figure 8 when $F_o=100$ kN. However, ideal responses can be achieved for both of the cases of $F_o$, equal to 100 and 700 kN.
by setting $P_{01}$ to 5 and 25 MPa respectively. In conclusion, the maximum allowable pressure determined by the design of the pneumatic cylinder must be high for large values of $F_{00}$.

The other parameters that influence the response are $\Delta F_s$ and $D_c$ which describe the weakening behavior of the sample. The main design parameter that controls the response during sample weakening is load capacity of the damper “$F_{dc}$”. Figure 9 illustrates the velocity response of the system to 4 cases of sample weakening. In general, higher $F_{dc}$ is required for samples with larger $\Delta F_s$ or smaller $D_c$, otherwise the response will have an overshoot. As shown in Figure 9, for $\Delta F_s=120$ kN and $D_c=10$ mm, a good response can be achieved when $F_{dc}=150$ kN. The response is still acceptable when either $D_c$ is decreased to 3 mm or $\Delta F_s$ is increased to 240 kN, which means that $F_{dc}$ is high enough to provide the required damping force. However, if $D_c$ decreases to 3 mm and $\Delta F_s$ increases to 240 kN simultaneously, the damper cannot provide enough resisting force and response will have a large overshoot (Figure 9). Therefore, when $D_c$ is small and $\Delta F_s$ is large, $F_{dc}$ should be high enough in order to achieve the desired velocity-step.

**Workspace of the apparatus**

As discussed in the previous section, the PPCV can be adjusted for different sample behaviors to achieve the desired response. However, in practice, there are some design constraints that restrict these parameters. The constraints are mainly on the maximum allowable pressure in the pneumatic cylinder and the load capacity of the damper which impose limits on $P_{01}$ and $C_d$. The design constraint on $A^*$ does not produce any restriction, in other words, the exhaust size can be built even larger than required. So, in this study, only restrictions introduced by maximum allowable pressure ($P_{max}$) and damping load capacity ($F_{dc}$) are presented.

For any given $P_{max}$ and $F_{dc}$, one can find in what range of $F_{00}$, $\Delta F_s$ and $D_c$ the apparatus is able to perform tests and achieve the desired velocity-step response. This range defines the workspace of the apparatus. In each case of the parametric search done for obtaining the workspace, a combination of sample parameters ($F_{00}$, $\Delta F_s$, $D_c$) are considered. For each of these combinations, a search should be done over different values of allowable PPCV ($P_{01}$, $A^*$, $C_d$) to determine if the desired curve can be generated by any set of PPCV. If yes, this point belongs to workspace. For this purpose, at each point of PPCV space, the velocity response is compared to the velocity-step curve and the error is found using least squares method. In this study, the response is considered acceptable if the overall error is less than 10% for a 2 cm stroke. After completion of the search for the first set of ($F_{00}$, $\Delta F_s$, $D_c$), another search is started for the next set. For the next set of ($F_{00}$, $\Delta F_s$, $D_c$), we start the search for PPCV from a small neighborhood of ($P_{01}$, $A^*$, $C_d$) found for the previous set of sample properties. Because points of sample parameter space are close to each other, the new set of PPCV should not be much different. In this way, the PPCV can be found with smaller amount of search and the computational cost decreases considerably.

Figures 10-13 illustrate the results of the search done for four different combinations of $F_{dc}$ and $P_{max}$. In each figure, workspace is obtained for three values of $D_c$, workspace is the area to the left of curves. For each $D_c$ different combinations of $F_s$ and $\Delta F_s/F_{00}$ are considered where $0 \leq F_s \leq 1.5$ MN and $0 \leq \Delta F_s/F_{00} \leq 0.9$. We use a non-dimensional parameter “$AF_s/F_{00}$” for performing the search because the limit of $\Delta F_s$ changes with magnitude of $F_s$. It can be inferred from these figures that two types of boundaries define workspace. A schematic diagram of these two boundaries and workspace (for a specific $D_c$) is shown in Figure 14. The first boundary defines the upper side of the curves which depends mainly on the damping capacity $F_{dc}$. For the area to the right of this boundary, overshoot produces errors larger than desired (like Figure 9, when $\Delta F_s=240$ kN, $D_c=3$ mm). The second boundary defines the lower part of curves which mostly depends on $P_{max}$. The area to the right of this boundary corresponds to conditions at which $P_{max}$ is not large enough relative to $F_{00}$. In these cases, the piston velocity drops considerably (like Figure 8, when $F_{00}=700$ kN). In extreme cases of the latter condition, the velocity curve may drop even before getting to 1 m/s.
Figures 10-13 imply that “Boundary I” moves to right and workspace expands as $D_c$ increases. $D_c$ increase means that $\Delta F_s$ occurs over a longer displacement and sample weakening is less abrupt. The other parameter that can move Boundary I is $F_{dc}$. Increasing the damping capacity enlarges workspace by moving Boundary I to right. Boundary II is mostly a function of $P_{\text{max}}$. Increasing $P_{\text{max}}$ results in expanding workspace moving Boundary II to right. $P_{\text{max}}$ can move Boundary I as well when $D_c$ is large enough. For example, when $D_c$ is 20 mm, by increasing $P_{\text{max}}$ Boundary II moves to right considerably. The reason is that the existence of higher pressure in the pneumatic cylinder gives the system more flexibility of adjustment to disturbances like sample weakening. However, this is not the case when $D_c$ is very small, for example, when $D_c$ is 1 mm, increasing $P_{\text{max}}$ does not move Boundary II. That’s because for small $D_c$, the sample weakening occurs so abruptly that even a higher pressure cannot make a difference.

**Design requirements**

In order to find design requirements, minimum values of $P_{\text{max}}$ and $F_{dc}$ should be obtained such that workspace covers the range of expected sample behaviors. For our application, the apparatus needs to be able to perform experiments on samples with $F_{s0}$ (0-500 kN), $\Delta F_s/F_{s0}$ (0-90%) and $D_c$ (5-20 mm). This range of $F_{s0}$ means that we can have large values of normal stress as high as 100 MPa during friction experiments while samples have a contact dimensions of 4cm×5cm or larger. As it can be seen in Figure 12, workspace for a $D_c$ of 5 mm covers the required range of samples. So, the maximum load capability of the damper should be 300 kN and the pneumatic cylinder must be able to hold pressures up to 12 MPa. These design requirements are feasible, and thus the desired step-velocity can be achieved for rock samples.

**CONCLUSIONS**

Although, constitutive relations of rock friction successfully describe the effect of sliding rate, normal stress, and sliding history on the frictional strength at quasi-static sliding rates, there is uncertainty in the accuracy of these relations for intermediate and high sliding rates. In this paper,
we have analyzed an apparatus designed for performing high-speed friction experiments and expanding our understanding of rock frictional behavior at high normal stresses, sliding rates and accelerations. For studying rate-state constitutive equations of friction, imposing a step-like velocity change is desired. The governing equations of the loading system of the apparatus have been derived and its dynamics has been studied in order to investigate the capability of achieving the desired velocity step for the range of expected rock-sample behaviors. Since feedback control is not applicable due to the high speed of the operation, achieving 1 m/s velocity in about 2 millimeters of displacement and subsequently maintaining constant velocity becomes quiet challenging, especially when sample force is initially large and diminishes during the experiment. For our application, the expected initial strength of samples can be as high as 500 kN while the strength may drop by 0-90 % during an experiment and Δc may be as small as 5 mm.

In order to minimize the error between the actual and desired velocity-step for different cases of sample behavior, a parametric study has been done to understand the effects of different PPCV on response. It has been shown that the initial pressure needs to be high enough to avoid velocity drop after getting to 1 m/s. Also, damping load capacity should be large enough to overcome the disturbances caused by sample weakening during an experiment; otherwise large overshoot can be generated. As a result of this parametric study, the maximum allowable pressure in the pneumatic cylinder $P_{\text{max}}$ and the maximum load capacity of the damper $F_{\text{dc}}$ have been identified as the most important design parameters that determine the workspace of the apparatus.

The workspace of the apparatus has been defined as the range of sample parameters ($F_{\text{sb}}, \Delta F_{\text{r}}, D_{\text{c}}$), for which, the loading system is capable of achieving the desired velocity-step response. We find that the workspace of the apparatus is restricted by two boundaries. Boundary I is mostly controlled by $F_{\text{dc}}$. This boundary moves in a direction that expands the workspace if $F_{\text{dc}}$ is increased. Increasing $P_{\text{max}}$ can also be influential on Boundary I and enlarging the workspace if $D_{\text{c}}$ of the sample is not very small. Boundary II is mainly a function of $P_{\text{max}}$ and workspace extends if $P_{\text{max}}$ increases. The optimum design requirements of the loading system are obtained by finding minimum magnitudes of $P_{\text{max}}$ and $F_{\text{dc}}$ so that the range of desired test conditions and expected sample behaviors occur within the workspace of the apparatus. By employing this design, a unique range of normal stresses, sliding rates and accelerations similar to real conditions during an earthquake can be achieved that cannot be attained by any other friction test machines in operation today. Using the designed apparatus, which is under construction, new aspects of frictional behavior of rocks can be investigated and major improvements of current understanding of rate-state friction behavior of rocks will be achieved.

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